Lesson 16. The Points-After-Touchdown Problem

1 The problem

- In an NFL football game, after scoring a touchdown, a team is given the option to try for:
  - a **1-point conversion**: 1 extra point by a field goal from the 15-yard line, or
  - a **2-point conversion**: 2 extra points by advancing the ball into the end zone from the 2-yard line
- Whether to "go for 2" is a classic debate – a few discussions on the topic:
  - https://theringer.com/nfl-two-point-conversions-pittsburgh-steelers-mike-tomlin-65d47282d853
  - https://fivethirtyeight.com/features/more-nfl-teams-are-going-for-two-just-as-they-should-be/
- Adding to the debate: in 2015, 1-point attempts were moved from the 2-yard line to the 15-yard line
- Conversion success rates from the 2014-2017 regular seasons (from http://www.pro-football-reference.com/):

	2014	2015	2016	2017
1-point conversion success rate	0.993	0.942	0.936	0.940
2-point conversion success rate	0.483	0.479	0.486	0.451

- Based on the current score and time remaining, should a team "go for 1" or "go for 2" in order to maximize the probability that it wins the game?
- How does the 2015 rule change affect a team's optimal conversion strategy?
- Let's try to answer these questions by modeling this problem as a stochastic dynamic program
- We will be roughly following this paper:

  H. Sackrowitz (2000). Refining the point(s)-after-touchdown decision. *Chance* **13**(3): 29-34.

2 Data

- Two teams: A and B
  - Assume that we (the decision-makers) are Team A
- Suppose we have the following data:

$$
\begin{aligned}
T &= \text{total number of possessions} \\
p_n &= \Pr\{\text{1-pt. conv. successful for Team } n \mid \text{1-pt. conv. attempted by Team } n\} && \text{for } n = \text{A, B} \\
q_n &= \Pr\{\text{2-pt. conv. successful for Team } n \mid \text{2-pt. conv. attempted by Team } n\} && \text{for } n = \text{A, B} \\
b_1 &= \Pr\{\text{1-pt. conv. attempted by Team B}\} \\
b_2 &= \Pr\{\text{2-pt. conv. attempted by Team B}\} \\
t_n &= \Pr\{\text{TD by Team } n \text{ in 1 possession}\} && \text{for } n = \text{A, B} \\
g_n &= \Pr\{\text{FG by Team } n \text{ in 1 possession}\} && \text{for } n = \text{A, B} \\
z_n &= \Pr\{\text{no score by Team } n \text{ in 1 possession}\} && \text{for } n = \text{A, B} \\
r &= \Pr\{\text{Team A wins in overtime}\}
\end{aligned}
$$

- What is the relationship between b_1 and b_2?

- What is the relationship between t_n, g_n and z_n?

- What is the probability that Team B scores 0 after a touchdown?

3 The stochastic DP

- Stages:

$$\begin{aligned} t = 0, 1, \dots, T-1 \quad &\leftrightarrow \quad \text{end of possession } t \\ t = T \quad &\leftrightarrow \quad \text{end of game} \end{aligned}$$

  - For our purposes, a possession ends when a team scores (TD or FG), or loses possession without scoring

- States:

$$\begin{array}{lll} (n, k, d) \quad \leftrightarrow & \text{Team } n\text{'s possession just ended} & \text{for } n \in \{\text{A}, \text{B}\} \\ & \text{Team } n \text{ just scored } k \text{ points} & \text{for } k \in \{0, 3, 6\} \\ & \text{Team A is ahead by } d \text{ points} & \text{for } d \in \{\dots, -1, 0, 1, \dots, \} \end{array}$$

- Value-to-go function:

$$f_t(n, k, d) = \text{maximum probability that Team A wins when in state } (n, k, d) \text{ at the end of possession } t$$
$$\text{for } n \in \{\text{A}, \text{B}\}, k \in \{0, 3, 6\}, d \in \{\dots, -1, 0, 1, \dots\}$$

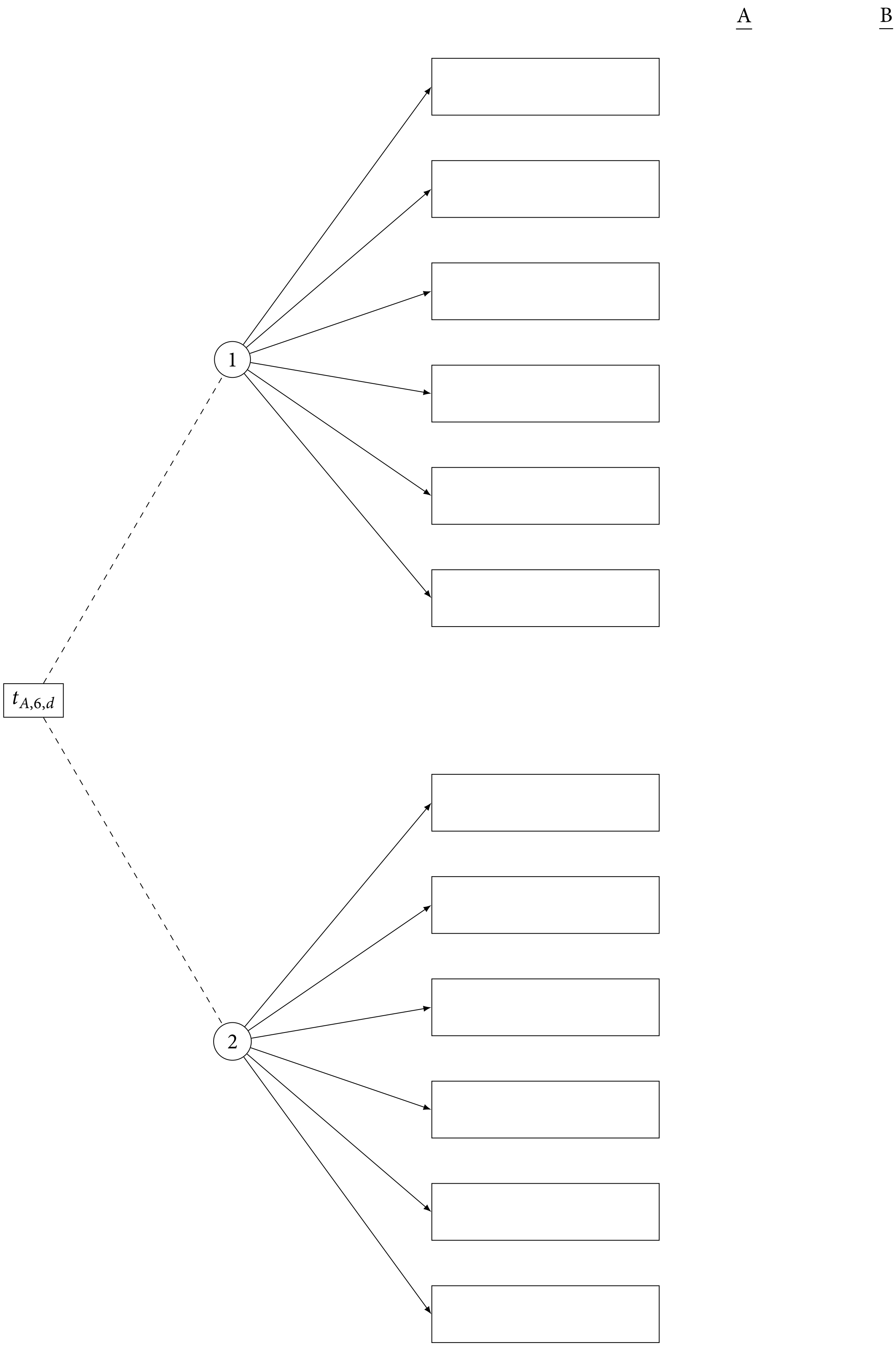
- Allowable decisions x_t at stage t and state (n, k, d):

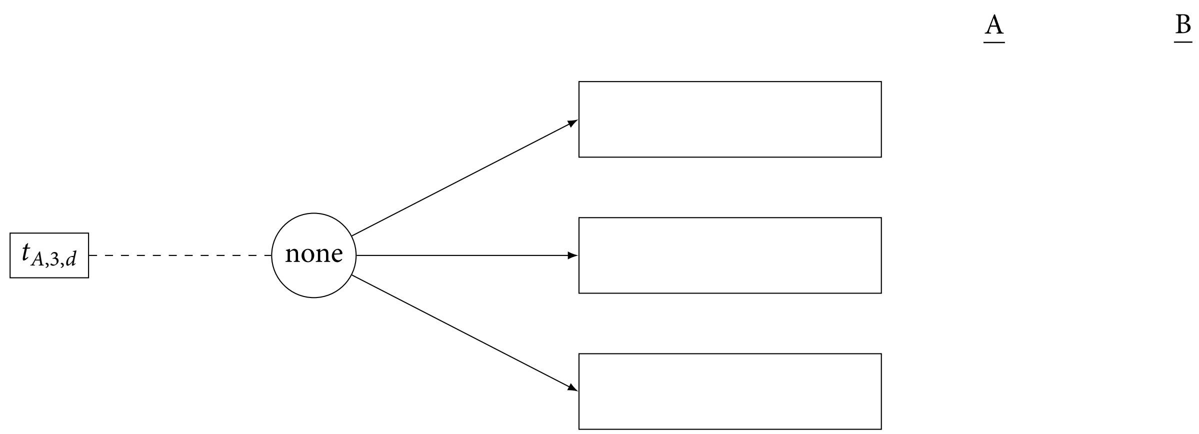
$$\begin{array}{ll} x_t \in \{1, 2\} & \text{if } n = \text{A and } k = 6 \\ x_t = \text{none} & \text{if } n = \text{A and } k \in \{0, 3\} \\ x_t = \text{none} & \text{if } n = \text{B and } k \in \{0, 3, 6\} \end{array}$$

- We need to consider transitions from the following states:

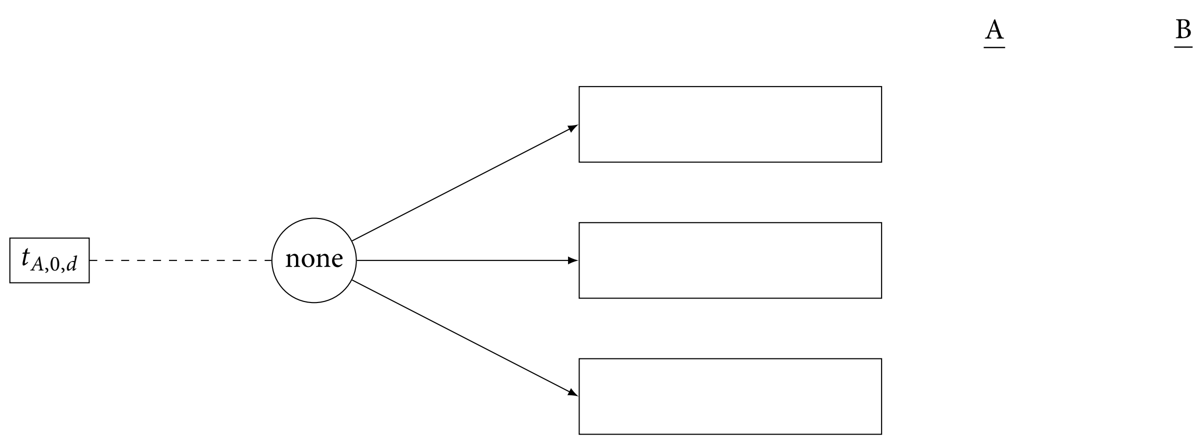
$$\begin{array}{ccc} (\text{A}, 6, d) & (\text{A}, 3, d) & (\text{A}, 0, d) \\ (\text{B}, 6, d) & (\text{B}, 3, d) & (\text{B}, 0, d) \end{array} \quad \text{for all } d$$

- Since our objective is to maximize the probability of winning, we set all the contributions in stages $t = 0, 1, \dots, T-1$ to 0, just like in the investment problem in Lesson 15

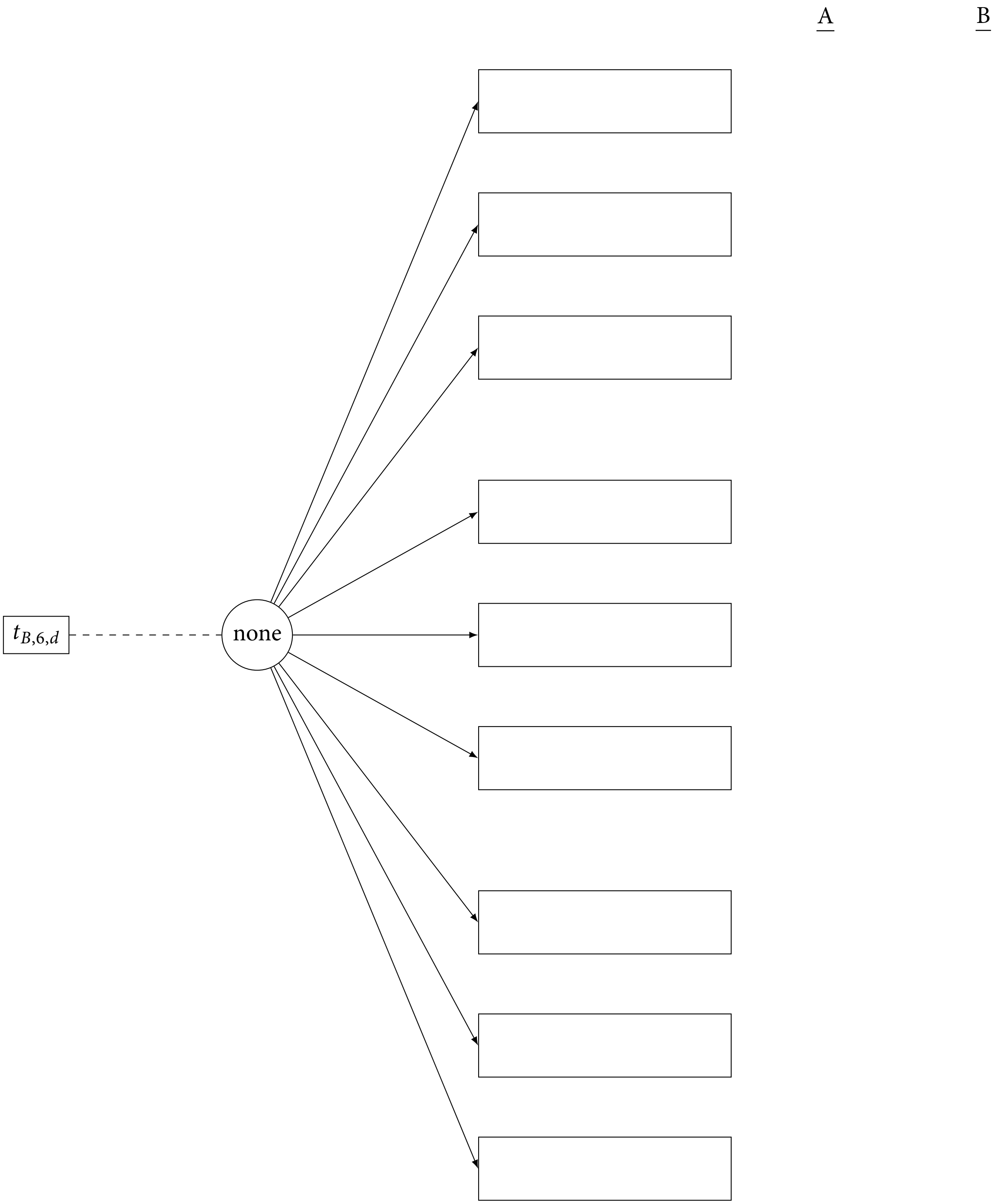

A
B
$t_{A,6,d}$
1
2

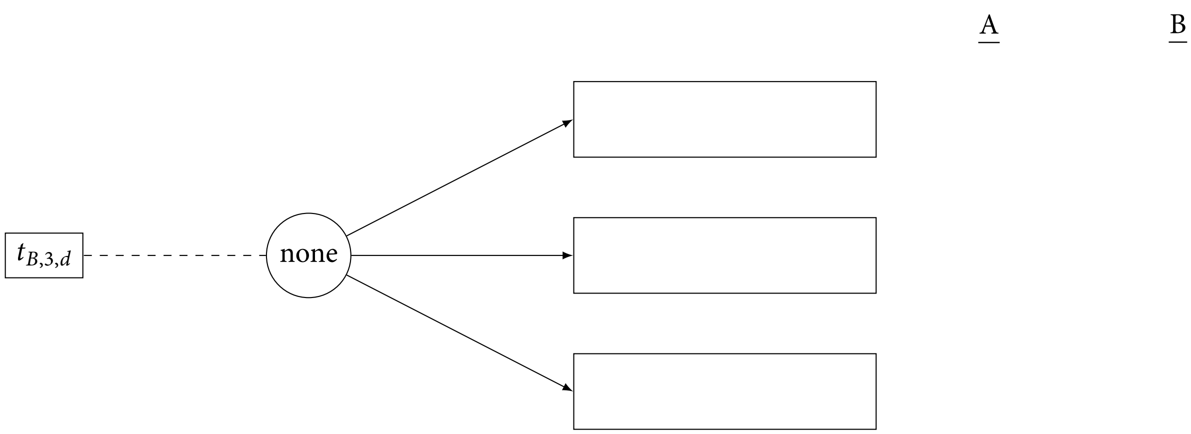


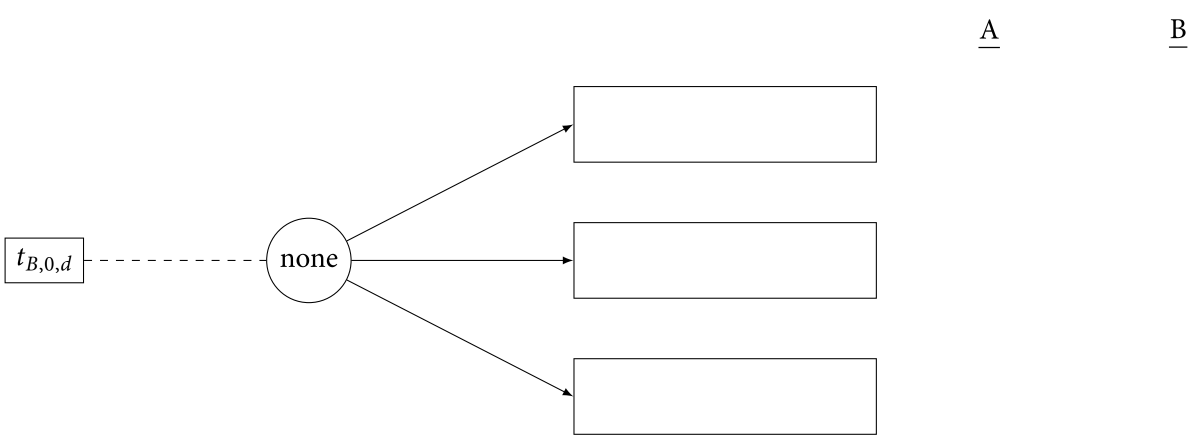
A
B
$t_{A,3,d}$
none



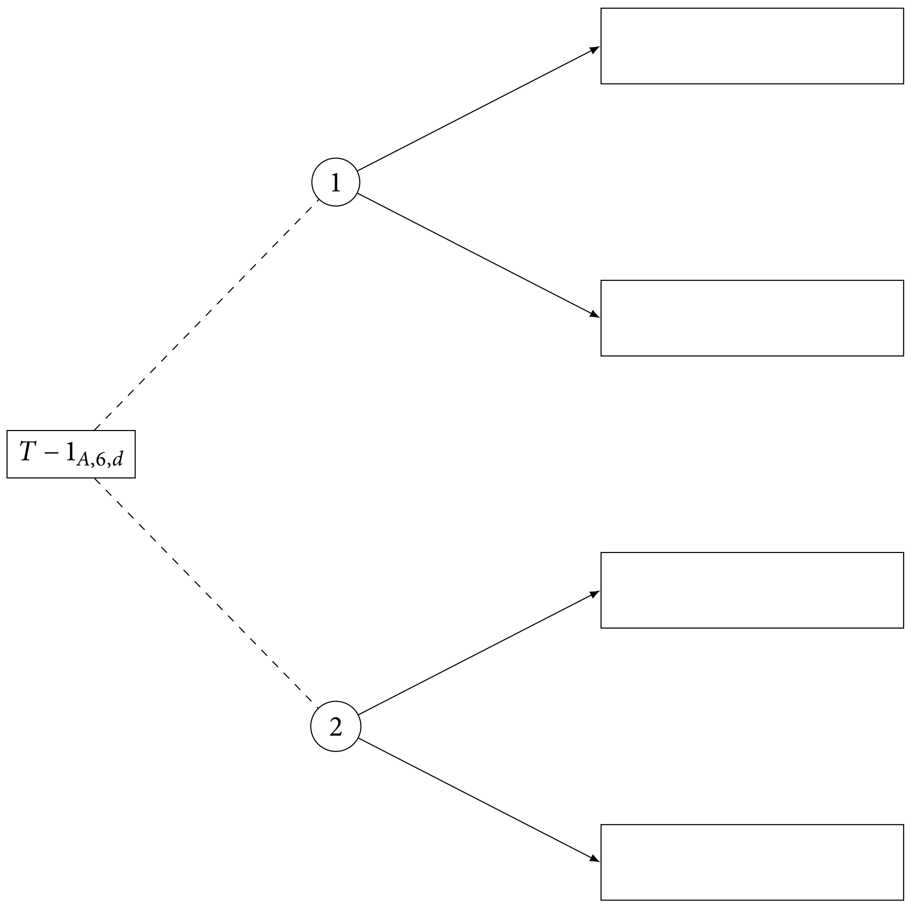
A
B
$t_{A,0,d}$
none


A
B
$t_{B,6,d}$
none


A
B
$t_{B,3,d}$
none

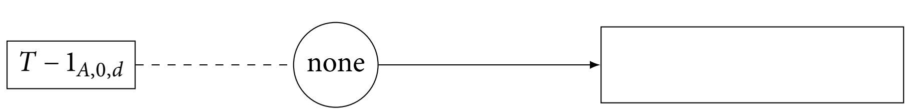

A
B
$t_{B,0,d}$
none

A

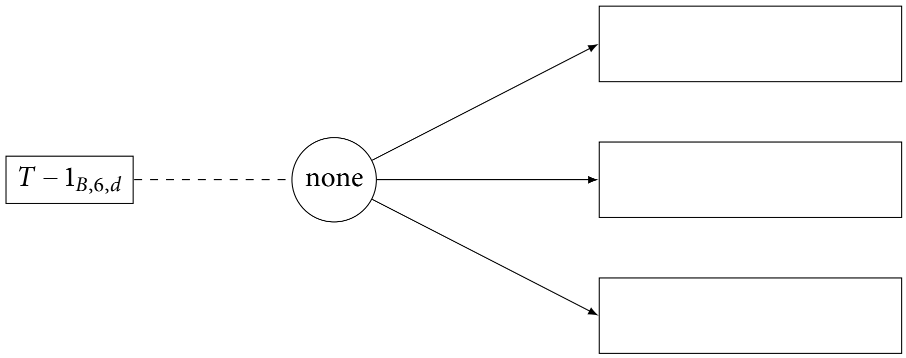


A

A



B



B

B